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Interface plasmon excitations of superlattices with defects

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Abstract. Interface plasmon excitations of superlattices with defects are investigated by the propagation matrix method. A dispersion relation is obtained and is shown to be of sufficient generality. The dispersion curves of the local modes are shown to be dependent upon the thickness and the dielectric constant of the defect layer. The obtained formula generalizes some earlier results obtained by other authors. Several special cases, including the limiting cases of quantum-well layered structures, are discussed and a new mode similar to the Giuliani and Quinn type mode is deduced.

1. Introduction

Superlattices (SL) are artificially fabricated crystals of alternate layers of different materials A and B with thicknesses d_1 and d_2. SL have recently attracted a lot of interest because of the technical advantages of depositing overlayers, such as molecular beam epitaxy (MBE). The study of excitations in SL produces new results [1–4]. Different kinds of mode can be encountered in SL systems: bulk optical or acoustic modes which are propagative or confined in the layers, interface modes which are mainly localized at the boundaries between the constitutive layers and surface modes whose frequencies lie in the bulk-mode energy gaps. In the case of plasmons for a perfect SL which consists of alternating layers of material, where one or both constituents contain free carriers, interface plasmon modes between the constitutive layers on adjacent interfaces couple through long-range Coulomb forces. Through use of Bloch's theorem, one then sees that a consequence of this coupling is a set of collective bulk plasmons of the whole SL structure characterized by a wavevector normal to the interfaces. Many researchers have done much work in the field of surface polaritons and surface plasmons [5–7]. Giuliani and Quinn [8] investigated the surface plasmon modes of a SL consisting of a periodic array of two-dimensional electron-gas (2DEG) layers embedded in a material of background dielectric constants ε_s. They showed the existence of surface plasmon modes (GQ modes) between two media of different dielectric constants if the wavevector is larger than some critical value. Bloss [9, 10] analysed the local plasmon modes between two semi-infinite SL and for the SL with one well doped with a different concentration. In analogy to the local phonon mode of vibrational lattices, they find that a local plasmon mode exists. Their discussions specialize the model to the case of the quantum-well widths going to zero, giving rise to the array of two-dimensional plasmon sheets. Surface plasmon modes of a SL consisting of metallic layers, which can be described

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by a local three-dimensional dielectric function, separated by insulating layers have been considered by Camley and Mills [11]. Related problems of acoustic and magnetic excitations in semi-infinite periodic structures have been investigated by Camley *et al* [12, 13]. As far as I know, in the frame of three-dimensional SL, the local modes similar to those discussed by Bloss have not been reported. In this paper, I give such a description by using the propagation matrix method. The local plasmon modes of SL with defects are investigated and the dispersion relations in closed analytic form are obtained. In a previous paper [14], the author has demonstrated that the propagation method can be used to derive the vibration modes of any SL system. I will illustrate the point further in this paper. The results of Bloss are shown to be the limiting cases of the present paper. Some new modes are presented.

2. Method and computation

2.1. The propagation method

The propagation matrix method has been proposed for computing the excitations and transmission and reflection of waves for layer systems in a previous paper [14]. The main ideas and formulae can be outlined as follows.

For any layered system, according to the boundary conditions of excitations in which one is interested, one may construct a state vector which must be continuous across the interfaces. In this paper, we are interested in a P-polarized electromagnetic wave. Therefore, this state vector should be $(E_T, H_T)^T$, with E_T, H_T standing for the tangential electric and magnetic fields at the interfaces. The propagation matrix of a film is defined as:

$$\boldsymbol{S}_1 = \boldsymbol{P}\boldsymbol{S}_2 \tag{1}$$

where $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ are the state vectors at either side of the film.

For a P-polarized electromagnetic wave, the P matrix of a film is:

$$P = \begin{bmatrix} \cosh(\alpha d) & \sinh(\alpha d)\varepsilon/\alpha \\ \alpha\sinh(\alpha d)/\varepsilon & \cosh(\alpha d) \end{bmatrix} \tag{2}$$

where $\alpha^2 = k^2 - \varepsilon\omega^2/c^2$. The matrix P of multilayers can be derived as

$$P = \prod_{i=1}^{N} P_i = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{3}$$

where N is the total layer number.

An intrinsic admittance Y is defined for bulk materials

$$Y = (\boldsymbol{H} \times \boldsymbol{n})/E^{\mathrm{T}} \tag{4}$$

where $\boldsymbol{n}$ is the unit vector in the direction of propagation. For the P-polarized wave, we have $Y = N_d/\cos(q_i)$ (q_i is the incidence angle, N_d is the refractive index of the dielectric). We define the effective admittance of any layered medium as following the same relations.

Assuming that the intrinsic admittance of a dielectric medium where the waves finally propagate is Y_d, then the effective admittance of a multilayer system is

$$Y_{\mathrm{eff}} = (P_{21} + Y_d P_{22})/(P_{11} + Y_d P_{12}) \tag{5}$$

while for semi-infinite SL, I have derived [14]

$$Y_{\text{eff}} = (P_{11} - \exp(-\alpha D))/P_{12} \tag{6}$$

where $\cosh(\alpha D) = (P_{11} + P_{22})/2$ and P is the propagation matrix of the unit cell of the SL.

Therefore:

$$R = \frac{(N/\cos(q_i) - Y_{\text{eff}})}{(N/\cos(q_i) + Y_{\text{eff}})} \tag{7}$$

where $N/\cos(q_i) = Y_0$ is the intrinsic admittance of the dielectric medium where the electromagnetic wave initially propagates. Y_{eff} is given by (5) for films and multilayers and by (6) for semi-infinite SL. Surface waves and bulk waves propagating along the surfaces and interfaces of any layer system correspond to:

$$Y_0 + Y_{\text{eff}} = 0 \tag{8}$$

where Y_0 is the intrinsic admittance of the dielectric cladding. Y_{eff} is the effective admittance of any layer system. Combining (5) with (8), we have:

$$Y_0 Y_d P_{12} + Y_0 P_{11} + Y_d P_{22} + P_{21} = 0 \tag{9}$$

where $Y_0 = N_0/\cos(q_i) = \alpha_0/\varepsilon_0$, $Y_d = N_d/\cos(q_d) = \alpha_d/\varepsilon_d$ for finite multilayers composed of N (where N is arbitrary) material.

$$\cos(qd) = \tfrac{1}{2}(P_{11} + P_{22}) \tag{10}$$

for infinite SL, and

$$(P_{11} - P_{22})Y_0 + P_{12}Y_0^2 - P_{21} = 0 \tag{11}$$

for surface waves in semi-infinite SL, where the P matrix contains all the information about the layer system. In (10) and (11), P is the propagation matrix of the unit cell of the SL. This scheme differs from the transfer matrix method [15–17] in that the T method can only be used to find solutions for infinite and semi-infinite SL while the P method can provide dispersion relations for arbitrary multilayer systems including finite and infinite SL. The P method can also be used to investigate the reflection and transmission of waves of any layered system. We have concluded [14], for any finite and infinite number of different layers of any materials in any configurations, that a large number of different kinds of modes can easily be investigated analytically and numerically using this scheme, showing the universality and generality of the method.

2.2. *P-polarization waves in superlattices*

For a SL ...ABABAB..., the P matrix of a 'unit cell' of the SL is

$$P = \begin{bmatrix} \cosh(\alpha_2 d_2) & \sinh(\alpha_2 d_2)\varepsilon_2/\alpha_2 \\ \alpha_2 \sinh(\alpha_2 d_2)/\varepsilon_2 & \cosh(\alpha_2 d_2) \end{bmatrix} \begin{bmatrix} \cosh(\alpha_1 d_1) & \sinh(\alpha_1 d_1)\varepsilon_1/\alpha_1 \\ \alpha_1 \sinh(\alpha_1 d_1)/\varepsilon_1 & \cosh(\alpha_1 d_1) \end{bmatrix} \tag{12}$$

with d_1 and ε_1 representing the thickness and dielectric constant of material A and d_2 and ε_2 of material B respectively.

$$\begin{aligned}
P_{11} &= \cosh(\alpha_1 d_1)\cosh(\alpha_2 d_2) + \frac{\alpha_1\varepsilon_2}{\alpha_2\varepsilon_1}\sinh(\alpha_1 d_1)\sinh(\alpha_2 d_2)\\
P_{12} &= \cosh(\alpha_1 d_1)\sinh(\alpha_2 d_2)\varepsilon_2/\alpha_2 + \sinh(\alpha_1 d_1)\cosh(\alpha_2 d_2)\varepsilon_1/\alpha_1\\
P_{21} &= \cosh(\alpha_1 d_1)\sinh(\alpha_2 d_2)\alpha_2/\varepsilon_2 + \sinh(\alpha_1 d_1)\cosh(\alpha_2 d_2)\alpha_1/\varepsilon_1\\
P_{22} &= \cosh(\alpha_1 d_1)\sinh(\alpha_2 d_2) + \frac{\alpha_2\varepsilon_1}{\alpha_1\varepsilon_2}\sinh(\alpha_1 d_1)\sinh(\alpha_2 d_2).
\end{aligned} \tag{13}$$

Therefore, the dispersion relation of a SL is obtained as follows:

$$\begin{aligned}
\cos(qL) &= \tfrac{1}{2}(P_{11} + P_{22})\\
&= \cosh(\alpha_1 d_1)\cosh(\alpha_2 d_2) + \frac{1}{2}\left(\frac{\alpha_2\varepsilon_1}{\alpha_1\varepsilon_2} + \frac{\alpha_1\varepsilon_2}{\alpha_2\varepsilon_1}\right)\sinh(\alpha_1 d_1)\sinh(\alpha_2 d_2)
\end{aligned} \tag{14}$$

where L is the SL periodicity. The dispersion of a surface mode localized in the near vicinity of the interface between material C and the semi-infinite SL is

$$\begin{aligned}
&(P_{11} - P_{22})\alpha/\varepsilon + P_{12}\alpha^2/\varepsilon^2 - P_{21} = 0\\
&\exp(-\beta L) = P_{11} + P_{12}\alpha/\varepsilon \qquad (\beta > 0)
\end{aligned} \tag{15}$$

where α and ε are the decaying constant of the surface wave in the dielectric C and the dielectric constant of C respectively. When $\varepsilon_B = \varepsilon_C = 1$, i.e. a semi-infinite stack of films from a surface active medium separated by material B with B cladding above, equation (15) reduces to the simple pairs of statements:

$$\alpha_1\varepsilon_2 = \pm\alpha_2\varepsilon_1 \qquad \text{and} \qquad \exp(-\beta L) = P_{11} + \alpha_2 P_{12}/\varepsilon_2. \tag{16}$$

We easily see that the upper sign corresponds to $\beta = (\alpha_1 d_1 + \alpha_2 d_2)/L$, an unacceptable value. However, if

$$\alpha_1\varepsilon_2 = -\alpha_2\varepsilon_1 \tag{17}$$

we have $\beta = (\alpha_1 d_1 - \alpha_2 d_2)/L$, which is acceptable if $\alpha_1 d_1 > \alpha_2 d_2$. The above results have been given by Camley and Mills [11] for non-retardation regions, which reduces to

$$\varepsilon_2 = -\varepsilon_1 \qquad \text{and} \qquad \beta = (d_1 - d_2)/L. \tag{18}$$

2.3. *Interface modes of superlattices with defects*

I further the research by investigating the SL ...ABABA©ABAB... where in one unit cell, the constituent B is substituted by material C. The P matrix of material C is

$$P = \begin{bmatrix} \cosh(\alpha d) & \sinh(\alpha d)\varepsilon/\alpha \\ \sinh(\alpha d)\alpha/\varepsilon & \cosh(\alpha d) \end{bmatrix}. \tag{19}$$

On both sides of material C is a semi-infinite SL composed of two components A and B. We assume that the semi-infinite SL have effective admittance Y_{eff}. Therefore from the multilayer dispersion relations in (9), we have

$$Y_{\text{eff}}^2\sinh(\alpha d)\varepsilon/\alpha + 2\cosh(\alpha d)Y_{\text{eff}} + \sinh(\alpha d)\alpha/\varepsilon = 0 \tag{20}$$

which reduces to

$$Y_{\mathrm{eff}} = \frac{\cosh(\alpha d) \mp 1}{\sinh(\alpha d)} \frac{\alpha}{\varepsilon} \tag{21}$$

i.e.

$$Y_{\mathrm{eff}} = -\tanh(\alpha d/2)\alpha/\varepsilon \tag{22}$$

or

$$Y_{\mathrm{eff}} = -\mathrm{cotanh}(\alpha d/2)\alpha/\varepsilon. \tag{23}$$

Let us first consider the simple case of an isolated slab of the dielectric C [18]. When we ignore retardation and set $Y_{\mathrm{eff}} = -k/\varepsilon_{\mathrm{vac}} = -k$, then

$$\varepsilon(\omega^+) = -\tanh(kd/2) \tag{24}$$

$$\varepsilon(\omega^-) = -\mathrm{cotanh}(kd/2) \tag{25}$$

which are the dispersions of a finite slab corresponding to the two modes of dielectric–vacuum interface. The two modes couple to produce an odd- and even-parity pair split by interaction between the two surfaces. Equations (22) and (24) are even-parity modes and (23) and (25) are odd-parity modes.

Then we consider the case of the symmetric SL-clad guide, i.e. SL with a defect layer. From (6) we obtain

$$Y_{\mathrm{eff}}^2 P_{12} + Y_{\mathrm{eff}}(P_{22} - P_{11}) - P_{21} = 0 \tag{26}$$

which is the implicit expression form of effective admittance of a semi-infinite SL. Thus substituting (13) into (26) we have the following general relations:

$$\begin{aligned} &\varepsilon_1^2[Y_{\mathrm{eff}}^2\varepsilon_2 \sinh(\alpha_1 d_1)\cosh(\alpha_2 d_2) + Y_{\mathrm{eff}}\sinh(\alpha_1 d_1)\sinh(\alpha_2 d_2)] \\ &\qquad + \varepsilon_1[Y_{\mathrm{eff}}^2\varepsilon_2^2 + 1]\cosh(\alpha_1 d_1)\sinh(\alpha_2 d_2) \\ &\qquad - [Y_{\mathrm{eff}}\varepsilon_2 \sinh(\alpha_2 d_2) - \cosh(\alpha_2 d_2)]\sinh(\alpha_1 d_1)\varepsilon_2 = 0 \end{aligned} \tag{27}$$

with the requirement $0 < \exp(-\beta L) = P_{11} - Y_{\mathrm{eff}} P_{12} < 1$.

Equation (27) is the main result of this paper. It describes the propagation of interface plasmon excitations in layered systems of semi-infinite SL or the SL with defects. In (27), $Y_{\mathrm{eff}} = -1/\varepsilon_0$ for semi-infinite SL and

$$Y_{\mathrm{eff}} = \frac{\cosh(\alpha_{\mathrm{d}}) \mp 1}{\sinh(\alpha_{\mathrm{d}})}\left(\frac{1}{\varepsilon}\right)$$

for SL with a defect layer. The wavenumber k is omitted from Y_{eff}.

In the following computations, we ignore retardation. Assuming that $\varepsilon_2 = \varepsilon$, we know that when $d \to \infty$, there exist surface modes provided that $d_1 > d_2$. When d is finite, we may regard the structure as two identical SL coupled by the dielectric C. Suppose $\varepsilon_1(\omega) = 1 - \omega_{\mathrm{P}}^2/\omega^2$ with $\omega_{\mathrm{P}} = 15$ eV. This corresponds to a model of aluminium. In figure 1 curves 1 and 2 show the dispersion of the coupled surface modes, where $d_2 = 0.5d_1$; $d = 1.5d_1$ in curve 1, $d = 0.75d_1$ in curve 2. We find that there exist two local modes when $d > d_2$ and $d_1 > d_2$. The smaller the separation between the SL, the larger the coupling. There are no local interface modes between the two coupled SL when $d < d_2$ and $d_1 > d_2$. When $d_2 > d_1$ and d is finite, there may exist a local mode provided that $d < d_2$. Curve 3 shows the results when $d_2 = 3d_1$ and $d = 2d_1$.

Figure 2 shows the dispersion curves of the local modes when $\varepsilon \neq \varepsilon_2$ and $d = d_2$. Curve 1 represents the symmetrical modes and curve 2 the antisymmetrical modes, where $\varepsilon_2 = 2$, the upper two curves correspond to $\varepsilon = 3$, and the lower curve to $\varepsilon = 1.5$. There is a cut-off wavenumber for antisymmetrical modes.

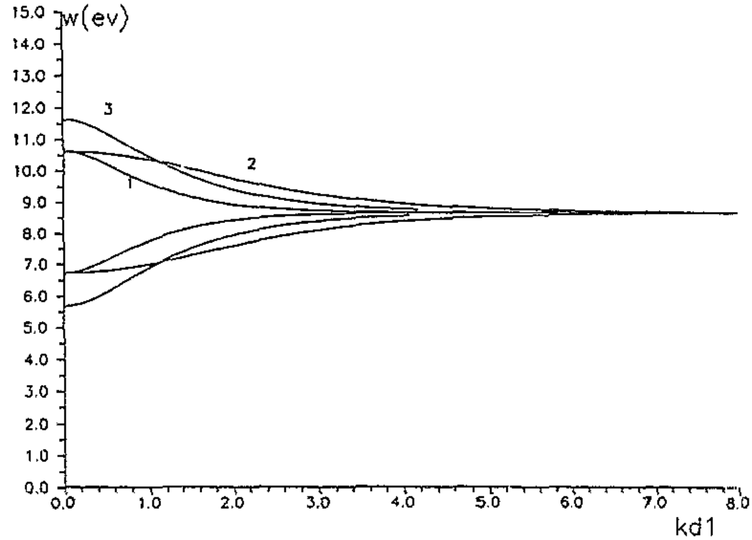

Figure 1. The dispersion curve of the local modes. Material A is aluminium. $\varepsilon = \varepsilon_2 = 2$, $d = 1.5d_1$ and $d_2 = 0.5d_1$ for curve 1; $d = 0.75d_1$ and $d_2 = 0.5d_1$ for curve 2; $d = 2d_1$ and $d_2 = 3d_1$ for curve 3.

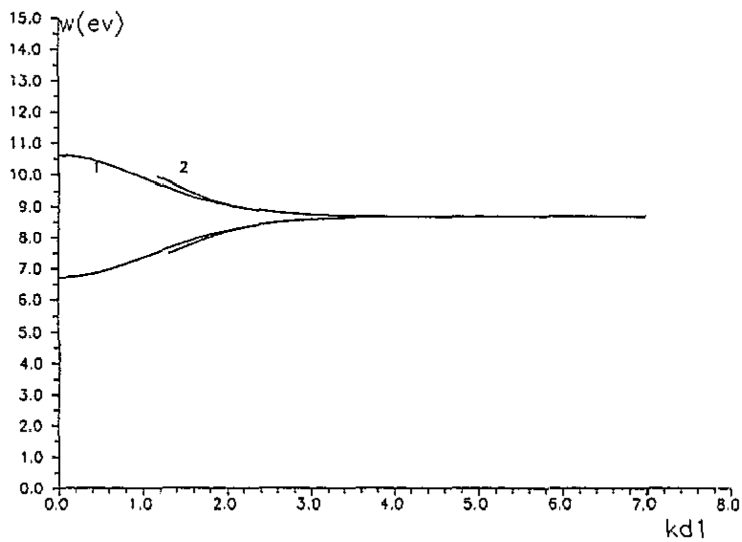

Figure 2. The dispersion curve of the local modes, which shows the influence of dielectric constant of the defect layer on the local modes of SL. Material A is aluminium. $d = d_2 = 2d_1$, $\varepsilon_2 = 2$, $\varepsilon = 3$ for the upper two curves, $\varepsilon = 1.5$ for the lower two curves. Curve 1 is the symmetrical mode while curve 2 is the antisymmetrical mode.

3. Discussion

It is useful to break this section into several parts. We initially consider that the separation between the SL is zero and then turn to SL consisting of a periodic array of doped quantum wells. It is shown that the collective surface modes presented by Camley and Mills [11] reduce to the famous Giuliani and Quinn plasmon modes (GQ modes) and our general dispersion relation (27) contains the local modes investigated by Bloss and some new modes in special cases.

3.1. ...ABABAABABA... structure

The above structure also corresponds to the two identical SL BABABA... separated by 2A. From (21), we know that $Y_{\rm eff} = 0$ or $Y_{\rm eff\infty}$. Substituting $Y = 0$ into (27), we obtain:

$$\varepsilon_1 \tanh(kd_2) = -\varepsilon_2 \tanh(kd_1) \qquad \text{and} \qquad \exp(-\beta L) = \cosh(kd_1)/\cosh(kd_2). \tag{28}$$

Equation (28) requires $d_1 < d_2$. Set $\varepsilon_2 = 1$ and $\varepsilon_1 = 1 - \omega_{\rm p}^2/\omega^2$, then

$$\omega^2 = \omega_{\rm p}^2/[1 + \tanh(kd_1)/\tanh(kd_2)]. \tag{29}$$

This is the dispersion relation of the even-parity mode in the sense that the electrostatic potential is even under reflection through the midpoint of film 2A. When $d_2 \gg d_1$, it is an optic-like mode, $kd \to 0$, $\omega = \omega_{\rm P}$.

In (27) setting $Y_{\rm eff} \to \infty$, we obtain

$$\varepsilon_1 \tanh(kd_1) = -\varepsilon_2 \tanh(kd_2) \tag{30}$$

which also requires $d_1 < d_2$. Set $\varepsilon_1 = 1$ and $\varepsilon_1 = 1 - \omega_{\rm p}^2/\omega^2$, then

$$\omega^2 = \omega_{\rm p}^2/[1 + \tanh(kd_2)/\tanh(kd_1)]. \tag{31}$$

The electrostatic potential has odd parity under reflection through the midpoint of the film. When $d_2 \gg d_1$, it is an acoustic-like mode, $kd \to 0$, $\omega = 0$. These results are similar to those of Fuchs and Kliewer [18] for the interface electrostatic excitations in isolated slabs. In the long-wavelength limit (compared to the interatomic distances) the interface vibrations of an ionic slab are composed of two surface modes: one symmetrical and one antisymmetrical, decaying exponentially into the interior of the slab. For $d_2 \gg d_1$, our results correspond exactly to an isolated slab result. Setting $d_2 = md_1$ when m decreases from a large value, the frequency of the optic-like mode will decrease and that the acoustic-like mode will increase in the small-wavenumber range, as shown in figure 3, which shows the effect of layered structure on a slab. The smaller m ($m > 1$) is, the smaller the gap between two modes.

3.2. Semi-infinite array of 2DEG

Many papers [8, 19–22] have investigated the plasmon modes of a semi-infinite array of layers located at $z = la$, where $l = 0, 1, 2 \ldots$. These layers are embedded in a medium of background dielectric constant $\varepsilon_{\rm s}$ and the space $z < 0$ is occupied by an insulator of dielectric constant ε_0. A new type of surface wave was reported by Giuliani and Quinn [8]. The GQ modes exist only for $\varepsilon_{\rm s} \neq \varepsilon_0$ and for wavelengths shorter than a critical value. This mode can be easily derived from (27). When A is a two dimensional electron gas, $kd_1 \to 0$ but we set

$$\varepsilon_1 \sinh(kd_1) = -\frac{\omega_{\rm p}^2}{\omega}\varepsilon k d_1 = -2\omega_{\rm 2D}^2 \varepsilon_2/\omega^2$$

where $\omega_{\rm P}^2 = 4\pi ne^2/m\varepsilon d_1$, $\omega_{\rm 2D}^2 = 2\pi ne^2 k/m\varepsilon$, $\varepsilon = \varepsilon_2$. (We assume that material A is simply material B with doping.)

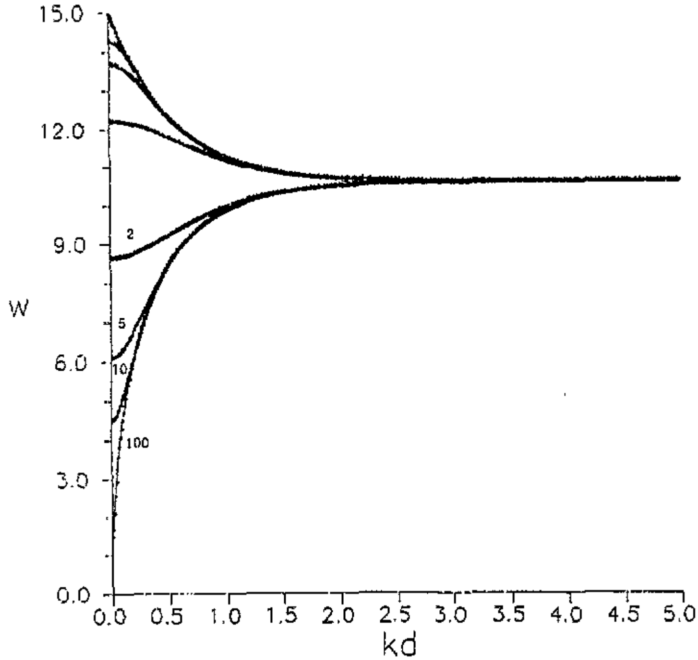

Figure 3. The vibration modes of a slab clad both sides by layered structures, showing the effect of a layered structure on a slab.

For a semi-infinite SL with insulator above, in (27) setting $Y_{\rm eff} = -1/\varepsilon_0$, we obtain the following explicit expressions:

$$\omega^2 = \frac{2\omega_{\rm 2D}^2[\varepsilon_2^2 \operatorname{cotanh}(kd_1) - \varepsilon_2\varepsilon_0]}{(\varepsilon_2^2 - \varepsilon_0^2)}. \tag{32}$$

Equation (32) is precisely the same as the dispersion of the GQ mode [8, 19].

If we say B is a 2DEG Many papers [8, 19–22] have investigated the plasmon modes of a semi-infinite array of and A is the bulk background dielectric medium with a metal or dipole active medium above, i.e. the first two-dimensional electron layer occurs a distance of the SL period from the interface, then $Y_{\rm eff} = -1/\varepsilon_0$. From (27) we have

$$\omega^2 = \frac{2\omega_{\rm 2D}^2[\varepsilon_1^2 \operatorname{cotanh}(kd_1) + \varepsilon_1\varepsilon_0]}{(\varepsilon_1^2 - \varepsilon_0^2)} \tag{33}$$

with $\exp(-\beta L) = \cosh(kd_1) + \varepsilon_0 \sinh(kd_1)/\varepsilon_1$, i.e. equation (33) requires $\varepsilon_0\varepsilon_1 < 0$ and $\tanh(kd_1/2) \leqslant |\varepsilon_0/\varepsilon_1|$.

The mode differs from the GQ mode in that it exists only for wavevectors smaller than some critical wavevector depending upon the ratio of the dielectric constant when $|\varepsilon_0| < |\varepsilon_1|$. When $|\varepsilon_0| > |\varepsilon_1|$ then we have

$$K^* = \frac{1}{d} \ln \frac{|\varepsilon_0| + \varepsilon_1}{|\varepsilon_0| - \varepsilon_1}$$

and the mode exists only when $k > k^*$. In the limit of strong coupling of the 2DEG, $kd_1 \to 0$, we have:

$$\omega^2 = \omega_0^2\varepsilon_1^2/(\varepsilon_1^2 - \varepsilon_0^2) \tag{34}$$

which is the collective charge-density oscillation modes of the 2DEG SL and in the limit of weak coupling, $kd \to \infty$, there exists a mode when $\varepsilon_0 = -\varepsilon_1$, which is the interface mode between dielectric media ε_0 and ε_1. In general cases, the mode is the coupling of the interface mode at the boundary of ε_0 and ε_1 with the vibration modes of the 2DEG SL.

3.3. Coupled quantum-well superlattices (QWS)

Bloss [9, 10] has recently investigated the coupled QWS in detail. Assume $d_2 \to 0$ and keep $kd_2\varepsilon_2 = -(2\omega_{2D}^2/\omega^2)\varepsilon_1$. The structure turns out to be a layered 2DEG Many papers [8, 19–22] have investigated the plasmon modes of a semi-infinite array of while the defect can be a quantum well doped with a different density of electrons. Thus from (21), we have

$$Y_{\text{eff}} = \frac{\omega^2}{\omega_{2D}^2}\frac{n}{n_0}. \tag{35}$$

Therefore from (27) we obtained the dispersion equations of an infinite periodic array of quantum wells where all the wells are doped with the same electron density n except for one well with the density n_0:

$$\left(\frac{\omega^2}{\omega_{2D}^2}\right)^2 - 2\left(\frac{\omega^2}{\omega_{2D}^2}\right)\text{cotanh}(kd_2) - \left(\frac{-2n}{n_0}+1\right)\left(\frac{n_0}{n}\right)^2 = 0$$

i.e.

$$(\omega^2/\omega_{2D}^2) = \text{cotanh}(kd_2) \pm \sqrt{\text{cotanh}(kd_2)^2 + \frac{n_0}{n}\left(\frac{n_0}{n} - 2\right)}. \tag{36}$$

Equation (36) is the same as equation (17) of [10]. It is found that when $n_0 \ll n$, the local mode is acoustic-like with velocity rather lower.

When material C is a bulk medium with dielectric constant ε, thickness d and $d_1 \Rightarrow 0$, but keeping $\varepsilon_1 \sinh(kd_1) = -(2\omega_{2D}^2/\omega^2)\varepsilon_2$, the structure is a coupled QWS separated by a bulk medium. From (27), we have:

$$\omega^2 = \frac{2\omega_{2D}^2[Y_{\text{eff}}^2\varepsilon_2^2\cosh(kd_2) + Y_{\text{eff}}\sinh(kd_2)]}{Y_{\text{eff}}^2\sinh(kd_2)\varepsilon_2^2 - \sinh(kd_2)} = \frac{2\omega_{2D}^2[Y_{\text{eff}}^2\varepsilon_2^2\text{cotanh}(kd_2) + Y_{\text{eff}}]}{Y_{\text{eff}}^2\varepsilon_2^2 - 1} \tag{37}$$

where Y_{eff} is given by (22) or (23).

When $\varepsilon = \varepsilon_2$, we have

$$\omega^2 = \omega_{2D}^2\sinh^2(kd/2)[-\text{cotanh}(kd_2) + \text{cotanh}(kd/2)] \tag{38}$$

$$\omega^2 = \omega_{2D}^2\cosh^2(kd/2)[\text{cotanh}(kd_2) - \tanh(kd/2)]. \tag{39}$$

These are the same as (32) of [9]. It should be noted that in this case, there are no dielectric constant discontinuities and these modes have no analogue in homogeneous media.

4. Conclusions

The general dispersion relations of SL with defects are derived by using the propagation matrix method. Explicit relations are obtained and are shown to be of sufficient generality. The results of some current papers including those of Bloss, Giuliani *et al* and Camley *et al* are the limiting cases of this paper. The dispersion curves are computed. It is found that two modes exist corresponding to a symmetrical and antisymmetrical combination of plasmon states of the individual SL. The modes exist only when $d > d_2$ and $d_1 > d_2$; or $d < d_2$ and $d_1 < d_2$ for $\varepsilon = \varepsilon_2$. Two special limits are discussed: (i) $d = 0$ (two identical SL of zero separation) there exist two modes corresponding to an acoustic-like mode and an optic-like mode when $d_2 \gg d_1$, which differs from two coupled semi-infinite SL of quantum wells [9]. In the latter case, only the symmetric eigenstate exists. (ii) Semi-infinite QWS and coupled QWS are discussed. A new mode similar to the GQ-type mode is deduced.

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